The Four-Step Method

The Four-Step Method is a systematic approach to problem solving that can be used for solving any problem. You may want to discuss these steps with your students before beginning work with the Teaching Problems. Understanding the purpose of each step can help students experience greater success.

Step 1: FIND OUT

The first step for students in solving a problem is to make sure they know what the problem is about and what they are being asked to find. Encourage them to try explaining the problem in their own words; this helps them better understand the information. They should ask themselves:

- What is happening in the problem?
- What do I have to find out to solve the problem?
- Are there any words or ideas I don't understand?
- What information can I use?
- Am I missing any information that I need?

Step 2: CHOOSE STRATEGIES

After students have identified what they are looking for and they know what information they have, they can make a plan for solving the problem. Now is the time to choose the strategy or combination of strategies that they think will be most helpful. They will find that there is often more than one way to solve the problem. In some cases, the problem may have to be broken down into smaller problems before the larger problem can be solved.

Step 3: SOLVE IT

Students now use the strategy or strategies they have chosen to solve the problem. It is very important that they record their work in a way that lets them see what they have completed. It is possible they will discover that the strategy they chose is not as helpful as they thought it would be. Emphasize that they should not be discouraged, but rather choose a different strategy and try again.

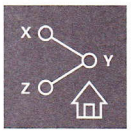
Step 4: LOOK BACK

After students have solved the problem, they should always check their answer by reading the problem again, looking back over each step, and checking their calculations. They should ask themselves:

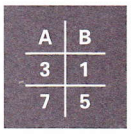
- Did I answer the question asked in the problem?
- Is more than one answer possible?
- Is my math correct?
- Does my answer make sense? Is it reasonable?
- Can I explain why I think my answer is correct?

Problem-Solving Strategies

Act Out or Use Objects



Use or Make a Picture or Diagram

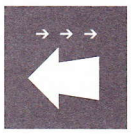


Use or Make a Table

Make an Organized List

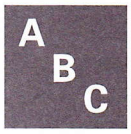
Guess and Check

Use or Look for a Pattern



Work Backwards

Use Logical Reasoning



Make It Simpler

Brainstorm

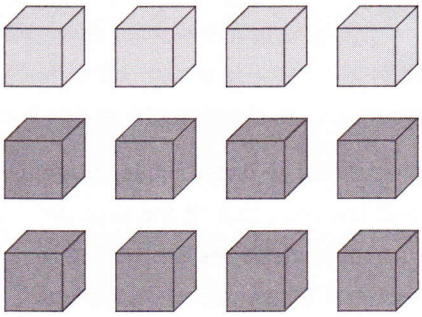
Ten Problem-Solving Strategies

The following ten strategies taught in *Problem Solver II* are useful for solving many kinds of problems. Students may discover other strategies on their own; when this happens, encourage them to share their discoveries with their classmates.

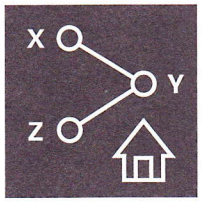
Act Out or Use Objects

Acting out a problem or having objects to move around gives students visual images of the problem and of the steps they must take to solve it. Using this strategy allows students to visualize arrangements, combinations, and relationships in the elements of a problem. Common manipulatives such as cubes, play money, and even scraps of colored paper work well to represent numbers of items and colors.

Example Jessica has 12 balloons. She has $\frac{1}{2}$ as many blue balloons as red balloons. How many balloons of each color does Jessica have?



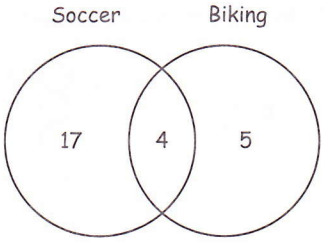
Using cubes to represent the 12 balloons, students can try different combinations until they find the one that gives Jessica half as many blue balloons as red.



Use or Make a Picture or Diagram

Some problems give a picture, diagram, or map as part of the data. For other problems, it may be helpful for students to draw their own picture or diagram. A simple pictorial representation can often help students understand and work with the data in the problem.

Example On Saturday, 21 scouts in all played soccer and 9 scouts in all went biking. Five of the scouts only biked. How many scouts did both? How many scouts were there altogether?



A Venn diagram would be helpful for solving this problem. After determining the numbers that belong in the three sections of the diagram, students could tell that there were 4 scouts who both played soccer and went biking, and 26 scouts in all.

Use or Make a Table

In some problems, students may need to use data from a table or a chart. In other problems, they may need to keep track of data in an orderly way. Making their own tables by listing key numbers in sequence can help students find missing data and discover or extend number patterns. This strategy is often used in combination with other strategies.

Example Devon is 1 year old and Darrin is 9. How old will they be when Darrin is twice as old as Devon?

	Age in Years							
Devon	1	2	3	4	5	6	7	8
Darrin	9	10	11	12	13	14	15	16

Creating a table for the problem data helps students see that when Darrin is 16 years old he will be twice as old as Devon, who will be 8 years old.

Make an Organized List

An organized list is a systematic way of recording a series of computations or exploring combinations of items. This strategy helps organize a student's thinking. A list makes it easy to see what has been done and to identify steps that still need to be completed. An organized list is especially helpful when a student wants to consider *all* the possibilities in order to find those that fit the problem.

Example There are more bicycles than cars in the park. There are 40 wheels in all. How many cars and how many bicycles can there be in the park?

Bicycles	Wheels	Cars	Wheels
1	2	1	4
2	4	2	8
3	6	3	12
4	8	4	16
5	10	5	20
6	12	6	24
7	14	7	28
8	16	8	32
⋮	⋮		
12	24		
⋮	⋮		
16	32		

After making an organized list of possibilities, students can review them to find the ones that fit the problem ("40 wheels in all"). As they extend the table, they will find three possible answers: 8 bicycles and 6 cars, or 12 bicycles and 4 cars, or 16 bicycles and 2 cars.

Guess and Check

Sometimes it is helpful to guess an answer when trying to solve a problem. When students use this strategy, they guess a number and then check to see if it fits with the other data given in the problem. If it doesn't, they decide whether their guess was too high or too low, then try to come closer with their next guess. They keep guessing and checking until they find a correct answer.

Example Mona has $\frac{1}{2}$ as many points as Jerry. Sam has 3 times as many points as Mona. Altogether they have 54 points. How many points does each friend have?

Guess 1 Jerry has 12 points
$\frac{1}{2}$ of 12 = 6 for Mona
3 x 6 = 18 for Sam
12 + 6 + 18 = 36, too low

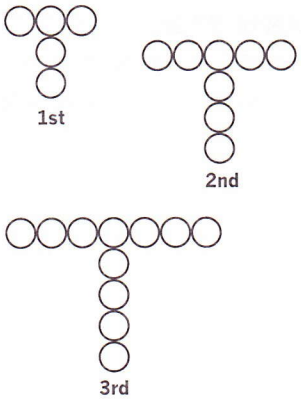
Guess 2 Jerry has 18 points
$\frac{1}{2}$ of 18 = 9 for Mona
3 x 9 = 27 for Sam
18 + 9 + 27 = 54

The student's first guess for Jerry's points gave a total that was too low. A higher guess leads to the correct total of 54, so the student now knows that Jerry has 18 points, Mona has 9, and Sam has 27.

Use or Look for a Pattern

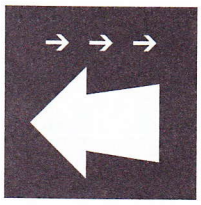
Looking for patterns is a very important strategy for problem solving; it is used to solve many different kinds of problems. A pattern occurs when a relationship is repeated again and again. A pattern may be numerical, visual, or behavioral. Identifying a pattern enables students to predict what will "come next" and what will happen again and again in the same way. In some problems, a pattern is given and students use it to solve the problem. In other problems, students must identify and extend the pattern in order to find a solution. Making a table often reveals patterns in data, so these two strategies are frequently used in combination.

Example Teyo hid a secret message in a field under a shape made of 35 stones. Trevor is looking for it. First he sees a T-shape made of 5 stones. Next he sees a T-shape made of 8 stones. Then he sees a T-shape made of 11 stones. The T-shapes keep getting larger and larger in the same way. Under which T-shape will Trevor find Teyo's secret message?



T	Number of stones
1st	5
2nd	8
3rd	11
4th	14
5th	17
⋮	⋮
11th	35

Placing information into a table will make it easier for students to see that the number of stones increases by 3 in each shape, and that the 11th T-shape will be made of 35 stones.



Work Backwards

To solve certain problems, it is necessary to begin with data presented at the end of the problem and then work backwards.

Example Kim has 4 times as many dimes as quarters, $\frac{1}{3}$ as many quarters as nickels, 2 times as many nickels as pennies, and 6 pennies. How much money does Kim have?

6 pennies	$ .06
2 × 6 = 12 nickels	$ .60
$\frac{1}{3}$ of 12 = 4 quarters	$ 1.00
4 × 4 = 16 dimes	$ 1.60
	$3.26

Starting with the last information given, 6 pennies, students would work backwards, using the clues to find how many of each type of coin Kim had. They would then add the values to get a total of $3.26.

Use Logical Reasoning

Logical reasoning is actually involved in all problem solving. However, certain problems include or imply conditional statements, such as: "if something is true, then..." or "if something is not true, then...." Problems like these require more formal logical reasoning as students step their way through the clues.

Example Use these clues to match Tanya, David, Angela, and Leah with their pets, which are a frog, a dog, a cat, and a hamster.

- Tanya is allergic to fur.
- David takes his animal for walks.
- Angela's pet chases birds in the yard.

	Frog	Dog	Cat	Hamster
Angela	No	No	Yes	No
David	No	Yes	No	No
Tanya	Yes	No	No	No
Leah	No	No	No	Yes

Using a logic chart, or matrix, will help students analyze the information in each clue. If Tanya is allergic to fur, could she have a frog? a dog? a cat? a hamster? If Tanya has a frog, can anyone else have the frog? Following this line of thinking, students write **Yes** or **No** in each cell of the chart until they see that Tanya has a frog, David has a dog, Angela has a cat, and Leah has a hamster.

Make It Simpler

Sometimes a problem can be made simpler by reducing large numbers to small numbers, or by reducing the number of items given in a problem. Having a simpler representation can make it easier to recognize the operation or process that can be used to solve the more complex problem. The simpler representation may even reveal a pattern that can be used to solve the problem.

Example Hubert has 7 bags of plastic forks, spoons, and knives. There are 24 forks, 24 spoons, and 24 knives in each bag. He needs to place 8 forks, 4 spoons, and 4 knives at each table. How many tables can Hubert set?

How many tables can Hubert set with just 1 bag of forks, spoons, and knives?

Students can simplify the problem by first finding out how many tables can be set from 1 bag (3 tables). From that, it is easy to figure out that with 7 bags, Hubert can set 21 tables.

Brainstorm

This strategy is often used when all else fails. When students cannot think of a similar problem they have solved before, and they cannot think of another strategy to use, brainstorming is a good strategy to try. It leads students to look at a problem in new and unusual ways. They should be encouraged to open up, stretch, allow for inspiration, be creative, be flexible, and keep on trying until a light goes on!

Example What could this code mean?

100 P i a D

parts pies paints

100 people in a _____

100 pounds in a _____

100 pieces in a _____

100 pennies in a _____ DOLLAR!

Although this would not be considered a true mathematics problem, and the brainstorming involves language skills, it requires creative thinking about quantitative relationships to figure out that 100 P i a D stands for 100 pennies in a dollar.